PHYS 100C, LECTURE #8

Thursday, April 23, 2009
12:07 PM

* Jefimenko Equations

"the other" Oleg

given ρ, J for all times, everywhere
find E and B:

$$V(r,t)=\frac{1}{4\pi\varepsilon_0}\cdot\int\frac{\rho(r',t_R)}{\mathscr{r}}\,d\tau'$$

$$A(r,t)=\frac{\mu_0}{4\pi}\int\frac{J(r',t_R)}{\mathscr{r}}\,d\tau'$$

$$E=-\nabla V-\frac{\partial A}{\partial t}\quad\text{and}$$

$$B=\nabla\times A$$

$$\nabla V=\frac{1}{4\pi\varepsilon_0}\cdot\int\left(-\frac{\dot{\rho}}{c}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}}-\rho\,\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

(see Lecture #7 Notes)

$$\frac{\partial A}{\partial t}=\frac{\mu_0}{4\pi}\cdot\int\frac{\dot{J}}{\mathscr{r}}\,d\tau'$$

$$E=\frac{1}{4\pi\varepsilon_0}\int\left(\frac{\rho}{\mathscr{r}^2}\hat{\mathscr{r}}+\frac{\dot{\rho}}{c\mathscr{r}}\hat{\mathscr{r}}-\frac{\dot{J}}{c^2\mathscr{r}}\right)d\tau'$$

Where ρ, $\dot{\rho}$ and $\dot{J}$ evaluated at t_R

Note that only the first term is Coulomb Eq. in retarded formalism, the terms with $\dot{\rho}$ and $\dot{J}$ are

"Unexpected"! ("from" naive approach)

Similarly, $B = \nabla \times A$

$$\nabla \times A = \frac{\mu_0}{4\pi} \int \left[\frac{1}{\tau} (\nabla \times J) - J \times \nabla\left(\frac{1}{\tau}\right) \right] d\tau'$$

(See $\nabla \times (f\vec{A})$ in "vector identities")

$\nabla \times J$ has terms like

$$\frac{\partial J_z}{\partial y} = \frac{\partial J_z}{\partial t_R} \cdot \frac{\partial t_R}{\partial y}$$

but $t_R = t - \tau/c \Rightarrow \partial t_R = -\frac{1}{c} \partial \tau$

$$\frac{\partial J_z}{\partial y} = \dot{J}_z \cdot \left(-\frac{1}{c}\right) \cdot \frac{\partial \tau}{\partial y}$$

these are now components of

$\dot{J} \times (\nabla \tau)$ (TRUST BUT VERIFY!)

$\nabla \tau = \hat{\tau}$ and $\nabla\left(\frac{1}{\tau}\right) = -\frac{\hat{\tau}}{\tau^2}$

$$B = \frac{\mu_0}{4\pi} \int \left(\frac{J}{\tau^2} + \frac{\dot{J}}{c\tau} \right) \times \hat{\tau} \cdot d\tau'$$

Hand-waving (naive) argument (also the best kind of argument!) would give us the first term for B in "retarded" time form, but not the $\dot{J}$ term.

Bottom Line: Time retarded ARGUMENT works for potentials V and A, but not for E & B. Expressions for E & B (derived from retarded potentials) include "non-obvious" contributions $\sim \dot{\rho}$ and $\dot{J}$

* Point Charges, Liénard-Wiechert Potentials

As before, $V(r,t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r, t_r)}{\mathscr{r}} d\tau'$

If we consider a single "point" charge q moving along some known trajectory $\vec{w}(t)$, then a naive person may decide that

$$\rho(r, t_R) = \delta^3(r - w(t_R)) \cdot q$$

and therefore

$$V(\mathscr{r}, t) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\mathscr{r}}$$

Where $\mathscr{r}$ is retarded distance from $\vec{w}(t_R)$ to observer.

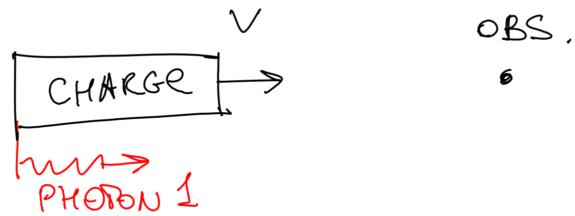
This is WRONG!

(bad day for naive people!)

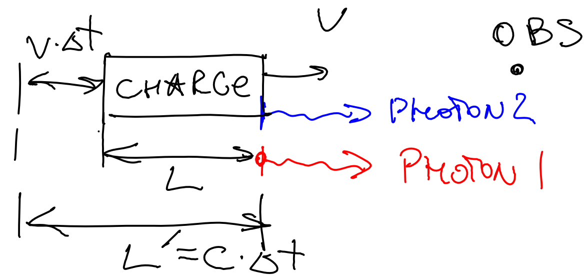
The charge is going to be larger than q, by a factor $(1 - \vec{v}\cdot\hat{\mathscr{r}}/c)^{-1}$

Why? Geometry/kinematics:

$t=0$
CARTOON:



$t=\Delta t$



Observer will "see" PHOTONS 1 & 2 at the same time and decide that actual "length" of charge is

$$L' = L + v\cdot\Delta t = c\cdot\Delta t$$

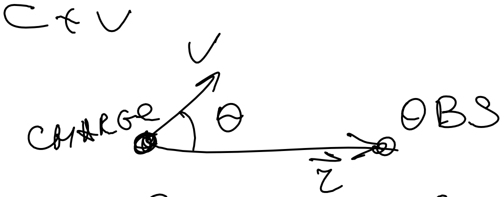
and since $\Delta t = \dfrac{L}{c-v}$

$$L' = L + \frac{Lv}{c-v} = \frac{Lc}{c-v}$$

IF CHARGE is moving away from observer,

$$L' = \frac{Lc}{c+v} \quad (\text{"shrinkage"})$$

Generally



$$L' = L\cdot\frac{c}{c - v\cdot\cos\theta} = L\cdot\frac{1}{1-\vec{v}\cdot\hat{z}/c}$$

The "volume" of charge and total

The "volume" of charge and total charge will therefore be multiplied by $\dfrac{1}{1-\vec{v}\cdot\hat{\mathscr{r}}/c}$

* Extreme CASE: $v=c$ and moving towards observer.
Observer won't see fields/photons, until ALL OF A SUDDEN they all come in at once! together with charge)

General Expression for V:

$$V(r,t)=\frac{1}{4\pi\varepsilon_0}\cdot\frac{qc}{\mathscr{r}c-\vec{\mathscr{r}}\cdot\vec{v}}$$

AND FOR A, similarly, since $\vec{J}=\rho\vec{v}$:

$$A(r,t)=\frac{\mu_0}{4\pi}\cdot\frac{qc\vec{v}}{\mathscr{r}c-\vec{\mathscr{r}}\cdot\vec{v}}=\frac{\vec{v}}{c^2}V(r,t)$$

These are Liénard-Wiechert potentials.

NEXT ON PHYS-100C:

Liénard-Wiechert MATH MASACRE!

(IT WILL BE LEGEN-DARY!)